



LIMPOPO

PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF
EDUCATION

NATIONAL
SENIOR CERTIFICATE

GRADE/GRAAD 12

MATHEMATICS P2/WISKUNDE V2

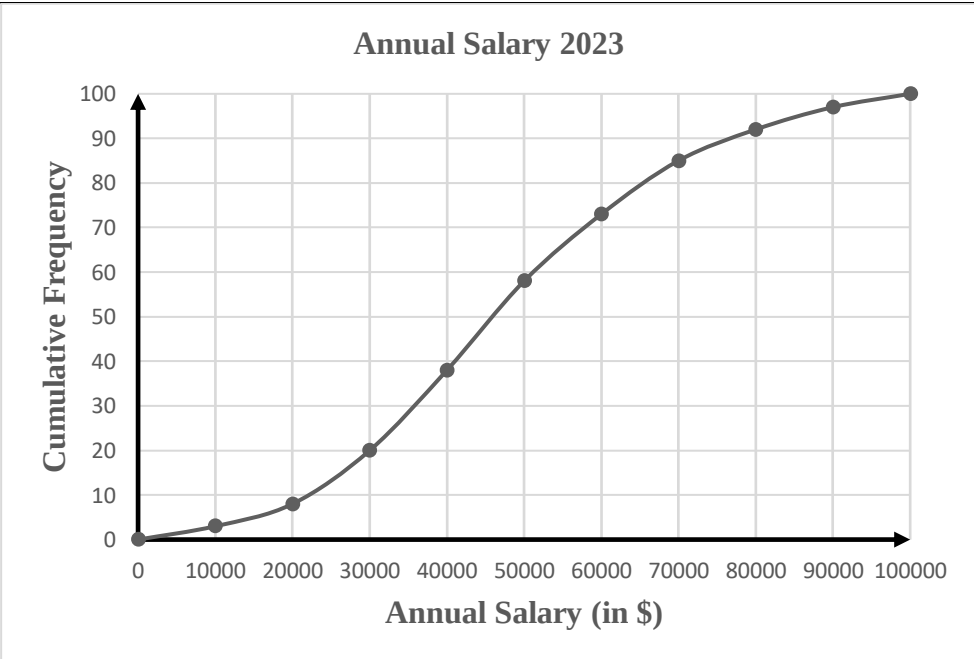
SEPTEMBER 2024

MARKING GUIDELINES/NASIENRIGLYNE

MARKS/PUNTE: 150

This marking guidelines consist of 15 pages/*Hierdie nasienriglyne bestaan uit 15 bladsye.*

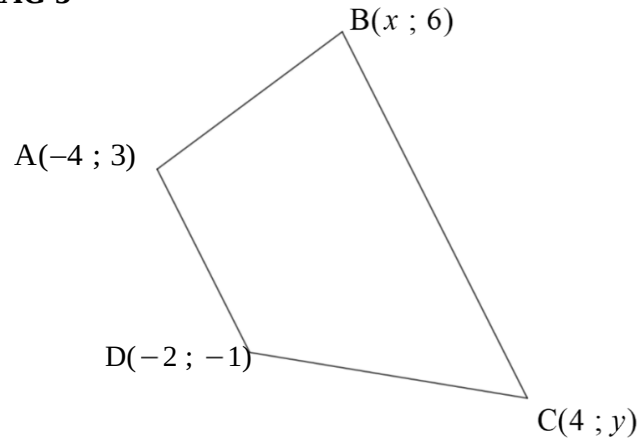
QUESTION/VRAAG 1

1.1	Annual salary range (in \$)/ <i>Jaarlikse salaris (in \$)</i>	Number of managers/ <i>Aantal bestuurders</i>	Cum Freq/ <i>Kum frekw</i>	✓39	(2)
	$0 \leq x < 10\,000$	3	3		
	$10\,000 \leq x < 20\,000$	5	8		
	$20\,000 \leq x < 30\,000$	12	20		
	$30\,000 \leq x < 40\,000$	19	39		
	$40\,000 \leq x < 50\,000$	20	59		
	$50\,000 \leq x < 60\,000$	14	73		
	$60\,000 \leq x < 70\,000$	12	85		
	$70\,000 \leq x < 80\,000$	7	92		
	$80\,000 \leq x < 90\,000$	5	97		
	$90\,000 \leq x < 100\,000$	3	100		
				✓100	
1.2	Annual Salary 2023 			✓(0 ; 0) ✓shape/ vorm ✓8 other points/ ander punte	(3)
1.3	$\text{IQR/IKV} = 61\,000 - 33\,000$ $= 28\,000$			✓accept/ aanvaar 61000 - 63000 ✓accept/ aanvaar 32000 - 34000 ✓answ/ antw	(3)

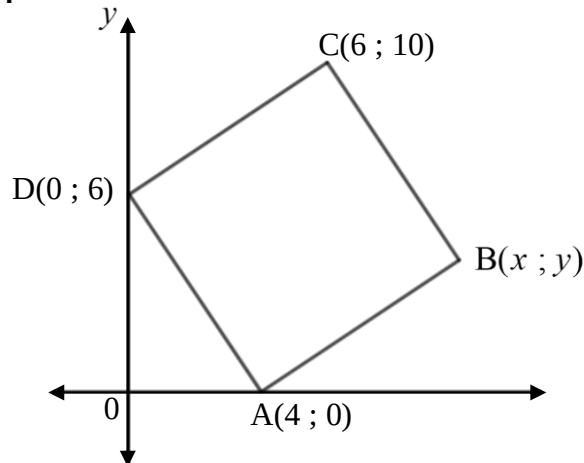
1.4	Less/Minder \$40 000: $\bar{x} = \frac{1055000}{39} = 27051,28$ After 15% increase/na 15% verhoging: $\bar{x} = 31108,97$ More than/Meer as \$40 000: $\bar{x} = \frac{3685000}{61} = 60409,84$ After 8% increase/na 8% verhoging: $\bar{x} = 65242,62$ New/nuwe: $\bar{x} = \frac{31108,97 \times 39 + 65242,62 \times 61}{100} = \frac{5034799,92}{100} = 50348$	✓ 27051,28 ✓ 31108,97 ✓ 65242,62 ✓ 50348	(4)
			[12]

QUESTION/VRAAG 2

2.1	$a = -26,21740146... = -26,22$ $b = 0,5389468765... = 0,54$ $\hat{y} = -26,22 + 0,54x$	✓ -26,22 ✓ 0,53 ✓ equation/vergelyking	(3)
2.2	$\hat{y} = -26,22 + 0,54(150) = 54,78$ OR/OF $\hat{y} = 54,62$ with calculator/ met sakrekenaar gebruik word	✓ substitution/vervanging ✓ answer/Antwoord OR/OF ✓✓ answer/antwoord	(2) (2)
2.3	As the girls get taller, they throw the ball further/ <i>Namate die meisies langer word, gaan hulle die bal verder gooi</i>	✓ answer/antwoord	(1)
2.4	No, the length of the girls cannot get longer indefinitely/ <i>Nee, die lengte van die meisies kan nie onbepaald langer word nie</i>	✓ No/nee ✓ reason/rede	(2)
			[8]

QUESTION/VRAAG 3

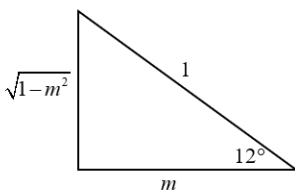
3.1	$m(AD) = m(BC)$ $\frac{3+1}{-4+2} = \frac{6-y}{x-4}$ $\frac{-2}{1} = \frac{6-y}{x-4}$ $-2x+8 = 6-y$ $y = 2x-2$	✓ $m(AD)$ ✓ $m(BC)$ ✓ equating/gelykstel ✓ equation/vergelyking	(4)
3.2	$BC = 2AD$ $\sqrt{(x-4)^2 + (6-y)^2} = 2\sqrt{(-4+2)^2 + (3+1)^2}$ $\sqrt{x^2 - 8x + 16 + 36 - 12y + y^2} = 2(2\sqrt{5})$ $x^2 - 8x + y^2 - 12y + 52 = 80$ $x^2 - 8x + y^2 - 12y = 28$	✓ subst in dist formula/ vervang in afst formule ✓ simplify/vereenvoudig ✓ 80 ✓ equation/vergelyking	(4)
3.3	$y = 2x - 2$① $x^2 - 8x + y^2 - 12y = 28$② Subst/vervang ① in ②: $x^2 - 8x + (2x-2)^2 - 12(2x-2) = 28$ $x^2 - 8x + 4x^2 - 8x + 4 - 24x + 24 - 28 = 0$ $5x^2 - 40x = 0$ $x^2 - 8x = 0$ $x(x-8) = 0$ $x = 0$ or/of $x = 8$ $y = -2$ $y = 14$	✓ substitution/vervanging ✓ simplify/vereenvoudig ✓ std form/std vorm ✓ factors/faktore ✓ values/waardes x ✓ values/waardes y	(6)
3.4	x translate 2 to left/transleer 2 links y translate 4 up/transleer 4 op	✓ 2 left/links ✓ 4 up/op	(2)
			[16]

QUESTION/VRAAG 4

4.1	4.1.1	B(10 ; 4)	✓ x ✓ y	(2)
	4.1.2	$DB = \sqrt{(0-10)^2 + (6-4)^2} = 2\sqrt{26}$	✓ subst in dist formula/ vervang in afstand formule ✓ answer/antwoord	(2)
	4.1.3	$\hat{C} = 90^\circ \therefore DB$ is a diameter (line subtend $90^\circ \angle$) E(5 ; 5) Radius = $\frac{1}{2}(2\sqrt{26}) = \sqrt{26}$ $(x-5)^2 + (y-5)^2 = 26$	✓ state DB is diameter/ stel DB as midlyn ✓ x ✓ y (midpt DB) ✓ radius = $\sqrt{26}$ ✓ LH ✓ RH / ✓ LK ✓ RK	(6)
	4.1.4	$m(\text{radius}) = \frac{6-5}{0-5} = -\frac{1}{5}$ $m(\text{tangent}) = 5$ $y = mx + c$ $y = 5x + c$ $6 = 5(0) + c$ $c = 6$ $y = 5x + 6$	✓ m(radius) ✓ m(tangent/raaklyn) ✓ subst m and point D/ vervang m en punt D ✓ equation	(4)

		OR/OF $m(\text{radius}) = \frac{6-5}{0-5} = -\frac{1}{5}$ $m(\text{tangent}) = 5$ $y - y_1 = m(x - x_1)$ $y - 6 = 5(x - 0)$ $y - 6 = 5x$ $y = 5x + 6$	✓ $m(\text{radius})$ ✓ $m(\text{tangent/raaklyn})$ ✓ subst m and point D / vervang m en punt D ✓ equation	(4)
	4.1.5	$m(\text{OE}) = \frac{0-5}{0-5} = 1$ $\tan \hat{\text{EOA}} = 1$ $\therefore \hat{\text{EOA}} = 45^\circ$ and/en $\hat{\text{DOA}} = 90^\circ$ $\therefore \text{EO bisect/halveer } \hat{\text{DOA}}$	✓ $m(\text{OE})$ ✓ $\tan$ def ✓ $\hat{\text{EOA}} = 45^\circ$	(3)
4.2	4.2.1	$(x-1)^2 + (y+1)^2 = 2(x-y)$ $x^2 - 2x + 1 + y^2 + 2y + 1 = 2x - 2y$ $x^2 - 4x + y^2 + 4y = -2$ $x^2 - 4x + 4 + y^2 + 4y + 4 = -2 + 4 + 4$ $(x-2)^2 + (y+2)^2 = 6$ $\therefore \text{Centre / midpunt } (2 ; -2)$	✓ simplify/vereenvoudig ✓ completing of square/ vierkantsvoltooing ✓ std form/std vorm ✓ x and/en y	(4)
	4.2.2	Radius = $\sqrt{6}$	✓ radius = $\sqrt{6}$	(1)
				[22]

QUESTION/VRAAG 5

5.1	5.1.1	$\cos(-12^\circ)$ $= \cos 12^\circ$ $= m$ 	✓ reduction/reduksie ✓ answer/antwoord	(2)
	5.1.2	$\cos 72^\circ$ $= \cos(60^\circ + 12^\circ)$ $= \cos 60^\circ \cos 12^\circ - \sin 60^\circ \sin 12^\circ$ $= \left(\frac{1}{2}\right)(m) - \left(\frac{\sqrt{3}}{2}\right)(\sqrt{1-m^2})$ $= \frac{m - \sqrt{3(1-m^2)}}{2}$	✓ $60^\circ + 12^\circ$ ✓ expansion/uitbrei ✓ substitution/vervangings	(3)
	5.1.3	$\cos 12^\circ = 2 \cos^2 6^\circ - 1$ $m = 2 \cos^2 6^\circ - 1$ $m + 1 = 2 \cos^2 6^\circ$ $\frac{m+1}{2} = \cos^2 6^\circ$ $\cos 6^\circ = \sqrt{\frac{m+1}{2}}$	✓ double $\angle$ formula/ dubbel $\angle$ formule ✓ equating to m/gelykstel m ✓ $\frac{m+1}{2}$ ✓ answer/antwoord	(4)
5.2		$\frac{\sin 234^\circ}{\cos 36^\circ} - \frac{\sin(x-90^\circ)\cos(90^\circ-2x)}{\sin x} = \cos 2x$ LHS = $\frac{\sin 234^\circ}{\cos 36^\circ} - \frac{\sin(x-90^\circ)\cos(90^\circ-2x)}{\sin x}$ $= \frac{-\sin 54^\circ}{\sin 54^\circ} - \frac{(-\cos x)(\sin 2x)}{\sin x}$ $= -1 - \frac{(-\cos x)(2 \sin x \cos x)}{\sin x}$ $= -1 + 2 \cos^2 x$ $= 2 \cos^2 x - 1$ $= \cos 2x$	✓ $-\sin 54^\circ$ ✓ $\sin 54^\circ$ ✓ $-\cos x$ ✓ $\sin 2x$ ✓ $2 \sin x \cos x$ ✓ $-1 + 2 \cos^2 x$	(6)
5.3	5.3.1	$\sin^2 B - \cos^2 B = 1$ $\cos^2 B - \sin^2 B = -1$ $\cos 2B = -1$	✓ -1	(1)
	5.3.2	$2B = 180^\circ - 0^\circ + k \cdot 360^\circ$ or $2B = 180^\circ + 0^\circ + k \cdot 360^\circ$ $B = 90^\circ + k \cdot 180^\circ, k \in \mathbb{Z}$ $B = 90^\circ + k \cdot 180^\circ, k \in \mathbb{Z}$ $\therefore \hat{B} = 90^\circ$	✓ both equations/beide vergelykings ✓ value/waarde $\hat{B}$	(2)

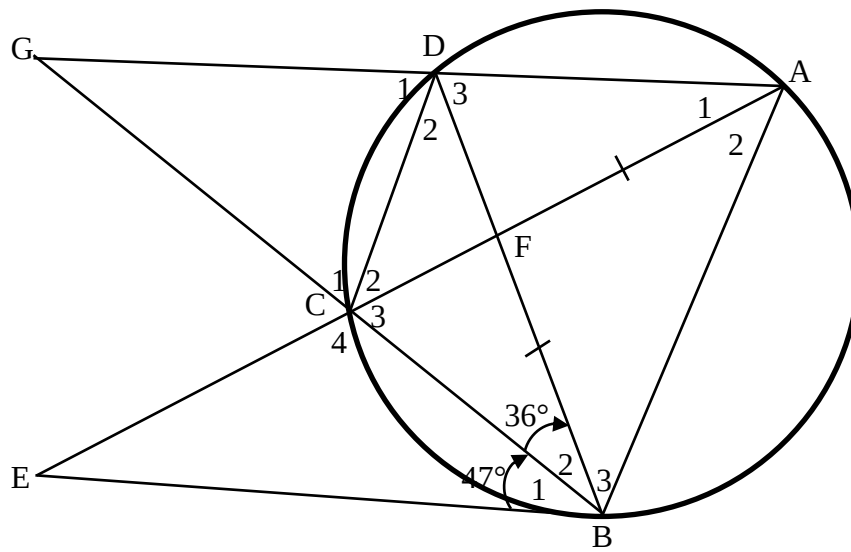
	5.3.3	$\sin(90^\circ - C) - \cos(90^\circ - C) = 0$ $\sin(90^\circ - C) = \cos(90^\circ - C)$ $\cos C = \sin C$ $1 = \frac{\sin C}{\cos C}$ $\tan C = 1$ $\hat{C} = 45^\circ + k.180, k \in \mathbb{Z}$ $\hat{C} = \{-135^\circ; 45^\circ\}$ OR/OF $\sin(90^\circ - C) - \cos(90^\circ - C) = 0$ $\sin(90^\circ - C) = \cos(90^\circ - C)$ $\cos C = \sin C$ $1 = \frac{\sin C}{\cos C}$ $\tan C = 1$ $\hat{C} = 45^\circ + k.360, k \in \mathbb{Z} \text{ or/of } \hat{C} = 180^\circ + 45^\circ + k.360$ $\hat{C} = 225^\circ + k.360, k \in \mathbb{Z}$ $\hat{C} = \{-135^\circ; 45^\circ\}$	$\checkmark$ substitution/vervangings $\hat{B}$ $\checkmark$ co-functions/kofunksies $\checkmark \tan C = 1$ $\checkmark$ 1 equation with $k.180^\circ$ /1 vergelyking $k.180^\circ$ $\checkmark -135^\circ \checkmark 45^\circ$ OR/OF $\checkmark$ substitution/vervangings $\hat{B}$ $\checkmark$ co-functions/kofunksies $\checkmark \tan C = 1$ $\checkmark$ 2 equations with $k.360^\circ$ /2 vergelykings $k.360^\circ$ $\checkmark -135^\circ \checkmark 45^\circ$	(6)
5.4		$P = 2 \cos x - \cos 2x$ $= 2 \cos x - (2 \cos^2 x - 1)$ $= -2 \cos^2 x + 2 \cos x + 1$ $\cos x = -\frac{b}{2a}$ $= -\frac{2}{2(-2)}$ $= \frac{1}{2}$ $P = -2 \cos^2 x + 2 \cos x + 1$ $= -2 \left(\frac{1}{2}\right)^2 + 2 \left(\frac{1}{2}\right) + 1$ $= \frac{3}{2}$	$\checkmark$ double/dubbel $\angle$ $\checkmark$ std form/std vorm $\checkmark$ substitution/vervangings $\checkmark \frac{1}{2}$ $\checkmark$ answer/antwoord	(5)
				[29]

QUESTION/VRAAG 6

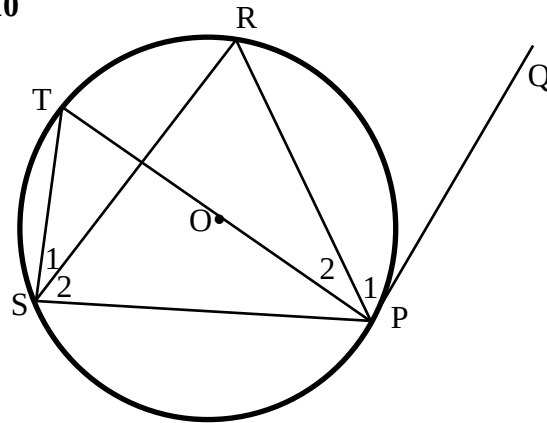
6.1	$a = 30^\circ$ $b = 1$		✓value/waarde a ✓value/waarde b	(2)
6.2	$f(x) = 2 \cos(0^\circ + 30^\circ)$ $f(x) = 2 \cos 30^\circ$ $f(x) = 2\left(\frac{\sqrt{3}}{2}\right)$ $f(x) = \sqrt{3}$		✓substitution/vervanging 0° ✓substitution/vervanging ✓answer/antwoord	(3)
6.3	6.3.1	$x = -135^\circ$ and/en $x = 45^\circ$	✓ -135° ✓ 45°	(2)
	6.3.2	$x \in [-90^\circ ; 0^\circ)$	✓critical values/kritieke waardes ✓notation/notasie	(2)
6.4	$f(x) = 2 \cos(x + 30^\circ - 30^\circ) = 2 \cos x$		✓ -30° ✓answer/antwoord	(2)
				[11]

QUESTION/VRAAG 7

7.1	$\hat{B}\hat{C}M = 90^\circ$	✓ 90°	(1)
7.2	$\hat{M}\hat{A}C = 90^\circ - \alpha$ $\hat{A}\hat{M}C = 180^\circ - (90^\circ - \alpha) - (90^\circ - \alpha)$ $\hat{A}\hat{M}C = 180^\circ - 180^\circ + 2\alpha$ $\hat{A}\hat{M}C = 2\alpha$	✓ $90^\circ - \alpha$ ✓ method/metode ✓ 2α	(3)
7.3	In $\triangle DBC$: $\tan \theta = \frac{h}{BC}$ $BC = \frac{h}{\tan \theta}$ In $\triangle ABC$: $\frac{AC}{\sin B} = \frac{BC}{\sin A}$ $\frac{AC}{\sin(180^\circ - 2\alpha)} = \frac{\frac{h}{\tan \theta}}{\sin \alpha}$ $AC = \frac{h}{\tan \theta} \times \frac{\sin 2\alpha}{1} \times \frac{1}{\sin \alpha}$ $AC = \frac{h \cdot 2 \sin \alpha \cos \alpha}{\tan \theta \sin \alpha}$ $AC = \frac{2h \cos \alpha}{\tan \theta}$	✓ tan def ✓ $\frac{h}{\tan \theta}$ ✓ substitute in sine rule/ vervang in sinreël ✓ $\sin 2\alpha$ ✓ $2 \sin \alpha \cos \alpha$	(5)
7.4	In $\triangle AMC$: $\frac{AM}{\sin C} = \frac{AC}{\sin M}$ $\frac{r}{\sin(90^\circ - \alpha)} = \frac{\frac{2h \cos \alpha}{\tan \theta}}{\sin 2\alpha}$ $\frac{r}{\cos \alpha} = \frac{2h \cos \alpha}{\tan \theta} \times \frac{1}{2 \sin \alpha \cos \alpha}$ $r = \frac{2h \cos \alpha}{\tan \theta} \times \frac{1}{2 \sin \alpha \cos \alpha} \times \frac{\cos \alpha}{1}$ $r = \frac{h \cos \alpha}{\tan \theta \sin \alpha}$ $r = \frac{h}{\tan \theta \tan \alpha}$	✓ subst is sine rule/ vervang in sinreël ✓ $\cos \alpha$ ✓ simplify/vereenvoudig	(3)
			[12]

QUESTION/VRAAG 8

8.1	$\hat{B}_2 = \hat{A}_1 = 36^\circ$	$\angle$'s in the same segment / $\angle$ 'e in dies segment	✓S ✓R	(2)
8.2	$\hat{B}_1 = \hat{A}_2 = 47^\circ$	tan-chord theorem/rklyn krd stelling	✓S ✓R	(2)
8.3	$\hat{C}_1 = \hat{A}_1 + \hat{A}_2 = 83^\circ$	ext $\angle$ of cyclic quad/buite $\angle$ kvh	✓S ✓R	(2)
8.4	$\hat{C}_2 = \hat{B}_3 = 47^\circ$	$\angle$'s in the same segment/ $\angle$ 'e in dies segment	✓S	(3)
	$\hat{C}_1 + \hat{C}_2 = \hat{C}_4$	vert. opp $\angle$'s / regoorst $\angle$ 'e	✓S/R	
	$\hat{C}_4 = 130^\circ$		✓S	
				[9]

QUESTION/VRAAG 10

10.1	Draw diameter POT and join TS/<i>trek midlyn POT en verbind TS</i> $\hat{P}_1 + \hat{P}_2 = 90^\circ$ radius $\perp$ tangent/<i>raaklyn</i> $\hat{S}_1 + \hat{S}_2 = 90^\circ$ $\angle$ in semi circle/<i>sirkel</i> $\hat{S}_1 = \hat{P}_2$ $\angle$'s in same segment/<i>dieselfde segment</i> $\therefore \hat{P}_1 = \hat{S}_2$	✓ constr/konstr ✓S ✓R ✓S/R ✓S/R	(5)
10.2			
10.2.1	In $\triangle ABC$ and/en $\triangle DBA$: (i) $\hat{A}_1 = \hat{D}$ tan-chord theorem/<i>rklyn koord stelling</i> (ii) $\hat{C}_2 = \hat{A}_2$ tan-chord theorem/<i>rklyn koord stelling</i> $\therefore \triangle ABC \parallel \triangle DBA$ 3 $\angle$'s OR/OF In $\triangle ABC$ and/en $\triangle DBA$: (i) $\hat{A}_1 = \hat{D}$ tan-chord theorem/<i>rklyn koord stelling</i> (ii) $\hat{C}_2 = \hat{A}_2$ tan-chord theorem/<i>rklyn koord stelling</i> (iii) $\hat{CBA} = \hat{ABD}$ 3rd $\angle$ $\therefore \triangle ABC \parallel \triangle DBA$	✓S/R ✓S ✓R ✓S/R ✓S ✓S	(3) (3)

	10.2.2	$\frac{AB}{DB} = \frac{BC}{AB} \quad \Delta's$ $\therefore AB^2 = DB \cdot BC$	✓S ✓R	(2)
	10.2.3	In $\triangle CBM$ and/en $\triangle BAN$: (i) $\hat{M} = 2\hat{A}_1$ midpt $\angle = 2 \times \text{circumf/omtrek } \angle$ $\hat{A}_1 = \hat{D}$ proven/bewys $\therefore 2\hat{A}_1 = 2\hat{D}$ $\hat{N} = 2\hat{D}$ midpt $\angle = 2 \times \text{circumf/omtrek } \angle$ $\therefore \hat{M} = \hat{N}$ (ii) $2\hat{B}_1 = 180^\circ - \hat{M}$ int $\angle$'s of Δ / binne $\angle$'e Δ $\therefore \hat{B}_1 = 90^\circ - \frac{\hat{M}}{2}$ $2\hat{B}_3 = 180^\circ - \hat{N}$ int $\angle$'s of Δ / binne $\angle$'e Δ $\therefore \hat{B}_3 = 90^\circ - \frac{\hat{N}}{2}$ But/maar $\hat{M} = \hat{N}$ proven/bewys $\therefore \hat{B}_1 = \hat{B}_3$ $\therefore \triangle CBM \triangle BAN$ 3 $\angle$'s OR/OF In $\triangle CBM$ and/en $\triangle BAN$: (i) $\hat{M} = 2\hat{A}_1$ midpt $\angle = 2 \times \text{circumf/omtrek } \angle$ $\hat{A}_1 = \hat{D}$ proven/bewys $\therefore 2\hat{A}_1 = 2\hat{D}$ $\hat{N} = 2\hat{D}$ midpt $\angle = 2 \times \text{circumf/omtrek } \angle$ $\therefore \hat{M} = \hat{N}$ (ii) $2\hat{B}_1 = 180^\circ - \hat{M}$ int $\angle$'s of Δ / binne $\angle$'e Δ $\therefore \hat{B}_1 = 90^\circ - \frac{\hat{M}}{2}$ $2\hat{B}_3 = 180^\circ - \hat{N}$ int $\angle$'s of Δ / binne $\angle$'e Δ $\therefore \hat{B}_3 = 90^\circ - \frac{\hat{N}}{2}$ But/maar $\hat{M} = \hat{N}$ proven/bewys $\therefore \hat{B}_1 = \hat{B}_3$ (iii) $\hat{M}\hat{C}\hat{B} = \hat{B}\hat{A}\hat{N}$ 3rd $\angle$ $\therefore \triangle CBM \triangle BAN$	✓S/R ✓S ✓S/R ✓S ✓R ✓S/R ✓S ✓S ✓S	(5)

	10.2.4	$\frac{CB}{BA} = \frac{BM}{AN} \quad \Delta's$ $\frac{CB}{BA} = \frac{R}{r}$ $\frac{CB^2}{BA^2} = \frac{R^2}{r^2}$ and $AB^2 = DB.BC$ proven/bewys $\frac{CB^2}{DB.BC} = \frac{R^2}{r^2}$ $\frac{CB}{DB} = \frac{R}{r^2}$	✓S/R ✓squaring/vierkant ✓substitute/vervang $AB^2 = DB.BC$	(3)
				[18]

TOTAL/TOTAAL : 150